The use of these conditions yields the transcendental frequency equation,

$$|a_{ij}| = 0 \qquad i, j = 1, \ldots, 4 \tag{12}$$

The frequency Eq. (12) is written separately for the two cases depending on symmetric or anti-symmetric mode condition.

Numerical Computations and Results

Computations were carried out for the symmetric modes and for the two types of end conditions mentioned in Cases 1 and 2. The ratio of the density of the face sheet material to that of the honeycomb core r_ρ is assumed to be 1/21. The ratio of moduli of rigidity of the face sheet to the core is taken as 30 for all calculations. Poisson's ratio of all the layers is assumed to be equal and of magnitude 0.3. Computations were carried out for three values of h/l which are 0.001, 0.003, and 0.005 and for four different values of c/l ranging from 0.01 to 0.04 at interval of 0.01. The frequencies were generated for first three symmetric modes by varying the ratio M/ml from zero to 4.0. Some of these results are shown in Figs. 2 and 3. These figures show the variation of Ω_n/Ω_0 against the mass ratio M/ml for two modes. The plot in Fig. 2 is for a simply supported beam while that in Fig. 3 is for fixed ended beam.

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Attainment of Jupiter Entry Shock Velocities

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Introduction

EFFORTS to develop shock tubes that simulate entry of a probe into the atmosphere of Jupiter have been under way for several years. Progress towards these goals has been described by Menard,[1] Dannenberg,[2] Livingston and Menard,[3] and Compton and Cooper.[4] The conical arc driver[1–3] has increased the attainable shock speed with 1.0 torr initial pressure from 15 to 34 km/sec and has allowed the simulation of some of the variables for Jupiter entry; for example, the temperature. Yet, performance has fallen short of the goal of 40 km/sec shock velocities in hydrogen-helium mixtures with initial pressures of 1.0 torr or greater. The Voitenko compressor driven shock tube[4] produced shock waves at greater than Jupiter entry velocity, but was unsuitable for sustained investigation because it destroyed itself during each firing. With the annular arc accelerator (ANAA) shock tube, 47 km/sec shock velocities have been achieved in 1.0 torr of hydrogen.

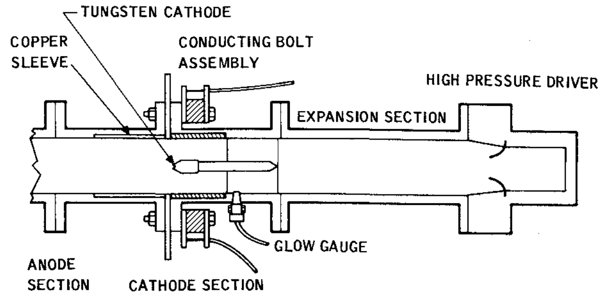

Fig. 1 Schematic diagram of the ANAA driver.

The ANAA consists of a cold gas driver, an expansion section, the electrode sections and the shock tube test section (see Fig. 1). A high pressure cold gas driver is used to flow a stream of gas past a high voltage electrode section. The driver gas accelerates into the expansion section driving a shock wave in front of it. As the flowing gas passes through the electrode sections, the energy of a high-voltage capacitor bank is discharged into the gas. The arc heated plasma immediately expands and cools, driving a shock wave down the tube. This immediate expansion greatly reduces the opportunity for the gas to lose energy by radiative cooling during the diaphragm opening process.

The initial operating experience of the ANAA shock tube has been previously described.[5] In the initial configuration, the capacitor bank lacked a switching capability and an auxiliary insulating diaphragm was placed between the electrode sections of the ANAA to act as a passive switch. However, the slow breaking of the insulating diaphragm disrupted the driver gas flow and prevented the ANAA from reaching its full potential.

Design

The ANAA driver is designed to coordinate the flow from a high pressure helium driver with the discharge from one hundred 20 KV capacitors. The capacitor bank is controlled by a high-voltage discharge system consisting of a flow detector, trigger generator, and a high-current switching system. Flow is initiated in the 15 cm diam electrode sections by bursting a scribed brass diaphragm with helium at a pressure of approximately 1500 psi. A shock wave followed by the driver gas then passes through the cathode section. The current is transferred to the gas through a pointed tungsten alloy cathode that is attached to an aluminum shaft. The aluminum anode section has a press-fit copper sleeve that absorbs the discharge current.

The ANAA driver switching system is capable of discharging over 1 million amp currents into the driver gas. The capacitor discharge trigger system includes a glow gage transducer to sense the arrival of the driver gas at the electrode section, a pulse forming circuit, a delay generator, a high voltage spark generator, and four spark gaps. The system is described in more detail elsewhere.[6,7]

Results

The effect on ANAA performance of variations in capacitor bank voltage, and the compositions and pressures of the driver and test gases has been investigated. The most significant variations in shock speed were produced by varying the test gas composition and the capacitor bank energy. In Fig. 2, a plot is shown of the shock velocity 8.3 m from the arc as a function of capacitor bank energy for several test gases. Velocities up to

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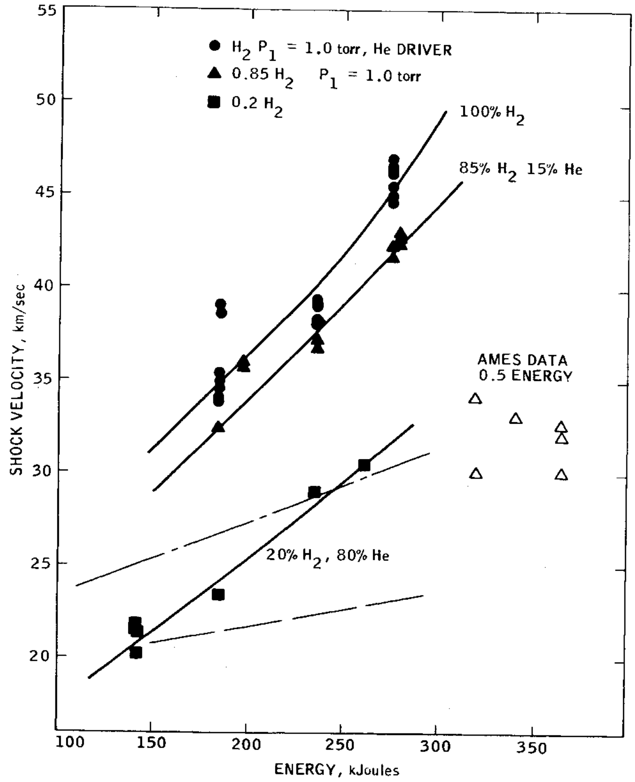

Fig. 2 Shock tube performance as a function of energy. The curves -.- and -- represent data from the JPL conical arc driver[3] with 0.5 torr, 10% H_2, 90% He test gas using hydrogen and helium driver gas, respectively.

47 km/sec have been achieved in 1.0 torr of hydrogen; however, the maximum velocity with 1.0 torr of 20% H_2, 80% He test gas was less than 32 km/sec. This effect may be due to the larger efficiency of hydrogen to absorb electrical energy than helium which is caused by hydrogen's higher effective resistance. These results indicate that the arc is discharging only into the shock heated test gas. Thus the driver gas acts only as a piston to compress and accelerate the test gas past the electrode sections.

The ANAA performance is compared to the performance of conical arc drivers in Fig. 2. The performance of the JPL conical arc shock tube[3] is shown for both helium and hydrogen driver gas. Recent data from the Ames Research Center's 1 Mjoule conical driver is also shown, and is plotted at one half the actual capacitor bank energy.[8] The ANAA shock velocities increase at a much greater rate with increasing energy than the conical arc drivers. This may be due to the reduced radiative cooling of the ANAA driver gas, although electromagnetic forces may also boost ANAA performance.

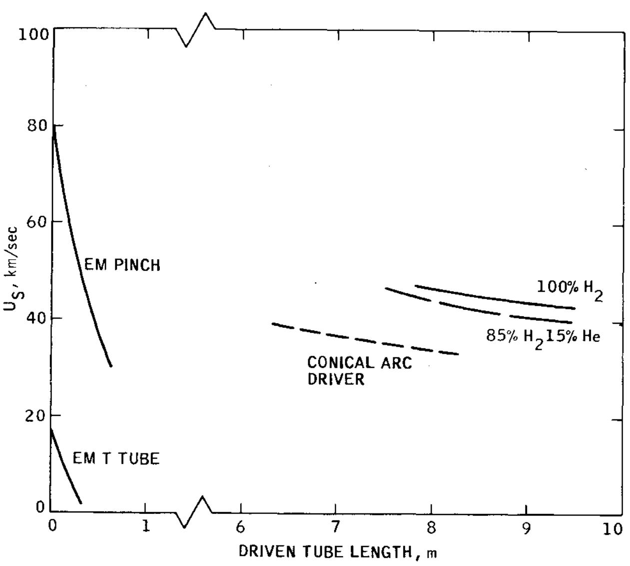

Fig. 3 Comparison of ANAA attenuation rates with other electric arc shock tubes (after Ref. 9).

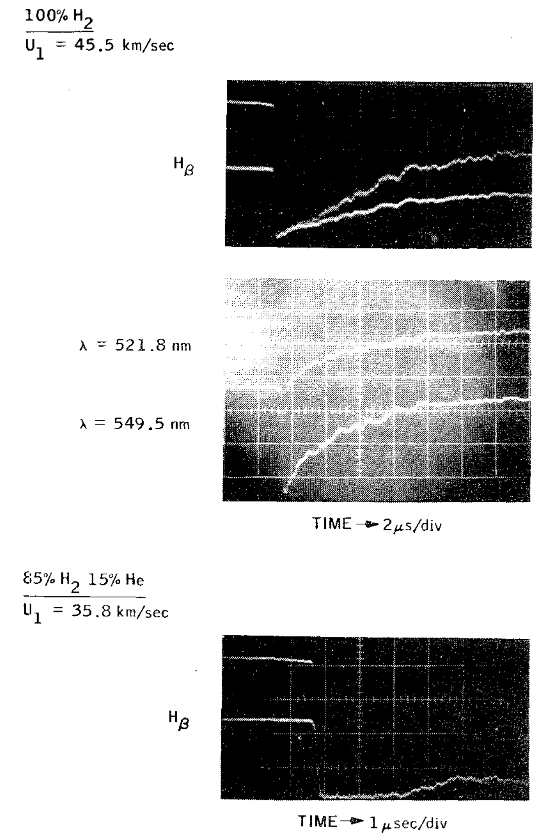

Fig. 4 Sample radiation emission measurements behind incident shock waves produced in the ANAA shock tube.

A comparison of the attenuation characteristics of the ANAA shock tube with other electric arc shock tubes is shown in Fig. 3 (after Ref. 9). The ANAA attenuation rate is not a function of test gas or capacitor bank voltage. Attenuation rates are very similar to those of the JPL conical arc driver[3] and contrast sharply with the blast wave-type decay characteristics of the electro magnetic-type shock tubes.

Radiative Emission Measurements

Measurements of the radiative emission behind the shock wave have been made to help determine the quality of the flow produced in the ANAA shock tube. A multi-channel spectrometer and optical system, previously described in Refs. 10 and 11, was used in this investigation.

The radiation relaxation signal changes characteristics with shock velocity. At the lower shock velocities the radiation signal undergoes a nonequilibrium rise in intensity similar to previous investigations,[10,11] reaches a steady plateau region, and then decays sharply indicating an end to the test time. However, for shock velocities greater than 40 km/sec the initial jump in radiative intensity is very rapid and is followed immediately by the decay of the continuum and line intensities. Examples of these characteristics are shown in Fig. 4. The decay in intensity behind the shock wave is due to radiative cooling of the 15,000 K shock heated gas, and makes it difficult to locate the end of the test slug. An attempt to detect the test slug boundary was made by introducing copper into the arc heated driver gas at the cathode and then measuring the radiative intensity of a copper spectral line behind the shock wave.

However, no contribution from copper was detected in the region behind the shock wave.

Data on test time has been obtained for runs where radiative cooling is unimportant. Values of test time between 2 and 5 μsec were obtained for the conditions covered. These results are approximately a factor of four less than calculations based on the theory of Mirels[12] and continue the trend of increasing divergence between experimental test time and one-dimensional theory for increasing shock velocity shown by Warren and Harris.[9] The test gas slug lengths obtained with the ANAA shock tube are comparable with results obtained with hydrogen helium mixtures in the conical arc driver shock tube,[1,10] although the test times are less due to the higher shock velocities obtained with the ANAA. While the test time has not been definitely measured at the highest speeds produced by the ANAA shock tube, initial analysis by Stickford[13] of the peak radiation and radiative decay rates observed behind the shock wave give good agreement with radiative cooling calculations and indicate that a usable test slug may be produced. Further measurements are needed, however, to show that the 2 μsec of test time needed to simulate the shock layer during Jupiter entry peak heating are obtained in the ANAA shock tube.

Conclusions

An annular arc accelerator shock tube has been built which produces shock velocities and pressures that simulate entry into the atmosphere of Jupiter. Shock velocities up to 47 km/sec have been produced in 1.0 torr of hydrogen which is considerably in excess of the 35 km/sec maximum velocity of conical arc drivers with three times the available energy. The attenuation rate with the ANAA shock tube was found to be comparable with the conical arc driver shock tube, and initial spectroscopic measurements indicate that an impurity free test slug is formed behind the shock wave.

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Adiabatic Ignition of Homogeneous Systems

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Introduction

THE thermal model of combustion of premixed gases is frequently employed in the evaluation of the rate of increase of temperature[1,2] and can adequately describe the reaction process in a homogeneous mixture of gases in a constant volume vessel with an initially uniform wall temperature T_o. Moreover if the effect of heat removal is neglected in the onset of explosion, an adiabatic analysis can be assumed.

Assuming an adiabatic process, the first law can be used to derive the following equation for the temperature[1]

$$dT/dt = Z(\bar{c}_v/Q)^{n-1}(T_f - T)^n e^{-E/RT} \tag{1}$$

where T, T_f, and t are dimensional temperature, final adiabatic flame temperature reached by the mixture and dimensional time, respectively. Also Z, Q, and E are the pre-exponential factor, molar efficiency of the fuel (cal per mole) and activation energy of chemical reaction (cal per mole), respectively, and are all characteristics of a particular reaction. Finally R, n, and $\bar{c}_v$ are the universal gas constant, effective order of reaction and mean heat capacity of the mixture at constant volume, respectively.

If Eq. (1) is nondimensionalized in the following manner,

$$\varepsilon = \frac{RT_o}{E}, \quad \theta = \frac{T - T_o}{\varepsilon T_o}, \quad \gamma = \frac{T_o}{T_f - T_o}, \quad \tau = \frac{Z(\bar{c}_v T_o/Q)^{n-1}t}{\varepsilon\gamma^n e^{1/\varepsilon}} \tag{2}$$

we obtain, as elsewhere,[4]

$$d\theta/d\tau = (1 - \varepsilon\gamma\theta)^n e^{(\theta/1+\varepsilon\theta)} \tag{3a}$$

As T is initially equal to T_o, θ must vanish at $\tau = 0$.

$$\theta(0) = 0 \tag{3b}$$

In the following, we shall discuss an approximate solution of Eq. (3) as well as develop an exact analytical solution of the equation. The approximate solution, obtained elsewhere[4] by means of a Zeldovich expansion and a subsequent application of the PLK method, will be shown to be not uniformly valid for all times of interest. The exact analytical solution, which has not been obtained previously, is the major result of this note.

Approximate Solutions of Eq. (3)

Inspection of Eq. (3) shows that the solution exhibits two limiting behaviors:

$$\varepsilon \to 0, \qquad \theta \text{ held fixed}, \qquad \theta \sim -\ln(1-\tau)$$

$$d\theta/d\tau = 0, \qquad \theta \sim 1/\varepsilon\gamma$$

The numerical integration of Eq. (3) has been conducted by Zeldovich and Voevodskii.[3]

Attempts to obtain an approximate solution to Eq. (3) have been based on the smallness of the parameter ε (ε is typically of the order of 10^{-2}). As a result, the well known Zeldovich expansion is frequently used

$$e^{(\theta/1+\varepsilon\theta)} \simeq e^{\theta(1-\varepsilon\theta+\varepsilon^2\theta^2-+\ldots)}, \quad \varepsilon\theta \ll 1 \tag{4}$$

Equation (3) is then approximated by

$$d\theta/d\tau = (1 - \varepsilon\gamma\theta)^n e^{\theta(1-\varepsilon\theta+\varepsilon^2\theta^2-+\ldots)} \tag{5}$$

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